

Problem 2.7

[Difficulty: 2]

2.7 A velocity field is given by $\vec{V} = ax\hat{i} - bty\hat{j}$, where $a = 1 \text{ s}^{-1}$ and $b = 1 \text{ s}^{-2}$. Find the equation of the streamlines at any time t . Plot several streamlines in the first quadrant at $t = 0 \text{ s}$, $t = 1 \text{ s}$, and $t = 20 \text{ s}$.

Given: Velocity field

Find: Equation for streamlines; Plot streamlines

Solution:

For streamlines
$$\frac{v}{u} = \frac{dy}{dx} = \frac{-b \cdot t \cdot y}{a \cdot x}$$

So, separating variables
$$\frac{dy}{y} = \frac{-b \cdot t}{a} \cdot \frac{dx}{x}$$

Integrating
$$\ln(y) = \frac{-b \cdot t}{a} \cdot \ln(x)$$

$$y = c \cdot x^{\frac{-b}{a} \cdot t}$$

The solution is

For $t = 0 \text{ s}$ $y = c$ For $t = 1 \text{ s}$ $y = \frac{c}{x}$ For $t = 20 \text{ s}$ $y = c \cdot x^{-20}$

t = 0

	c = 1	c = 2	c = 3
x	y	y	y
0.05	1.00	2.00	3.00
0.10	1.00	2.00	3.00
0.20	1.00	2.00	3.00
0.30	1.00	2.00	3.00
0.40	1.00	2.00	3.00
0.50	1.00	2.00	3.00
0.60	1.00	2.00	3.00
0.70	1.00	2.00	3.00
0.80	1.00	2.00	3.00
0.90	1.00	2.00	3.00
1.00	1.00	2.00	3.00
1.10	1.00	2.00	3.00
1.20	1.00	2.00	3.00
1.30	1.00	2.00	3.00
1.40	1.00	2.00	3.00
1.50	1.00	2.00	3.00
1.60	1.00	2.00	3.00
1.70	1.00	2.00	3.00
1.80	1.00	2.00	3.00
1.90	1.00	2.00	3.00
2.00	1.00	2.00	3.00

t = 1 s

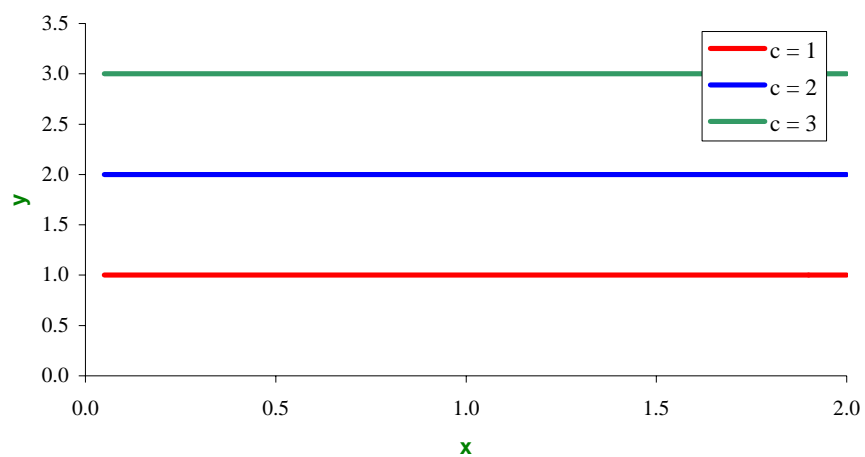
(### means too large to view)

	c = 1	c = 2	c = 3
x	y	y	y
0.05	20.00	40.00	60.00
0.10	10.00	20.00	30.00
0.20	5.00	10.00	15.00
0.30	3.33	6.67	10.00
0.40	2.50	5.00	7.50
0.50	2.00	4.00	6.00
0.60	1.67	3.33	5.00
0.70	1.43	2.86	4.29
0.80	1.25	2.50	3.75
0.90	1.11	2.22	3.33
1.00	1.00	2.00	3.00
1.10	0.91	1.82	2.73
1.20	0.83	1.67	2.50
1.30	0.77	1.54	2.31
1.40	0.71	1.43	2.14
1.50	0.67	1.33	2.00
1.60	0.63	1.25	1.88
1.70	0.59	1.18	1.76
1.80	0.56	1.11	1.67
1.90	0.53	1.05	1.58
2.00	0.50	1.00	1.50

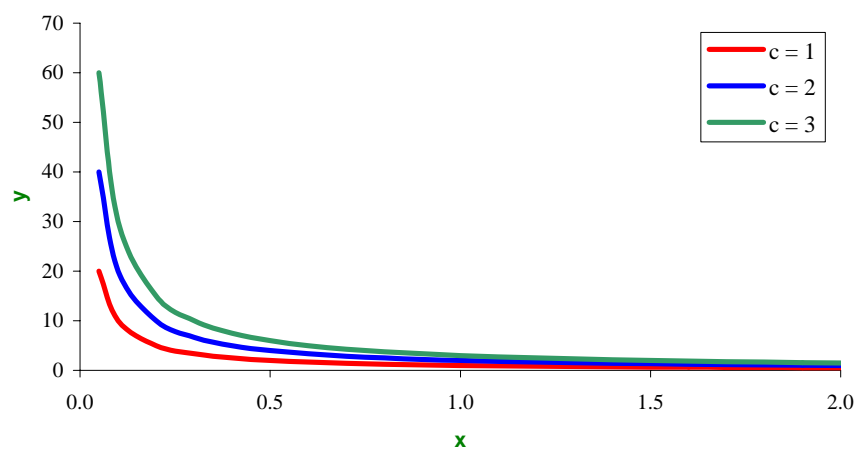
t = 20 s

	c = 1	c = 2	c = 3
x	y	y	y
0.05	#####	#####	#####
0.10	#####	#####	#####
0.20	#####	#####	#####
0.30	#####	#####	#####
0.40	#####	#####	#####
0.50	#####	#####	#####
0.60	#####	#####	#####
0.70	#####	#####	#####
0.80	86.74	173.47	260.21
0.90	8.23	16.45	24.68
1.00	1.00	2.00	3.00
1.10	0.15	0.30	0.45
1.20	0.03	0.05	0.08
1.30	0.01	0.01	0.02
1.40	0.00	0.00	0.00
1.50	0.00	0.00	0.00
1.60	0.00	0.00	0.00
1.70	0.00	0.00	0.00
1.80	0.00	0.00	0.00
1.90	0.00	0.00	0.00
2.00	0.00	0.00	0.00

Streamline Plot ($t = 0$)



Streamline Plot ($t = 1$ s)



Streamline Plot ($t = 20$ s)

